

**Exercise 26**Find  $y'$  and  $y''$ .

$$y = \ln(1 + \ln x)$$


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**Solution**Let  $u = 1 + \ln x$ .

$$y = \ln u$$

Take the derivative of the function with respect to  $x$  by using the chain rule.

$$\begin{aligned} y' &= \frac{d}{dx}(\ln u) \\ &= \frac{du}{dx} \frac{d}{du}(\ln u) \\ &= \left[ \frac{d}{dx}(1 + \ln x) \right] \left( \frac{1}{u} \right) \\ &= \left( \frac{1}{x} \right) \left( \frac{1}{1 + \ln x} \right) \\ &= \frac{1}{x(1 + \ln x)} \end{aligned}$$

Take another derivative by using the quotient rule and the product rule.

$$\begin{aligned} y'' &= \frac{d}{dx} \left[ \frac{1}{x(1 + \ln x)} \right] \\ &= \frac{\left[ \frac{d}{dx}(1) \right] [x(1 + \ln x)] - \left\{ \frac{d}{dx}[x(1 + \ln x)] \right\} (1)}{[x(1 + \ln x)]^2} \\ &= \frac{(0)[x(1 + \ln x)] - \frac{d}{dx}[x(1 + \ln x)]}{x^2(1 + \ln x)^2} \\ &= -\frac{1}{x^2(1 + \ln x)^2} \frac{d}{dx}[x(1 + \ln x)] \\ &= -\frac{1}{x^2(1 + \ln x)^2} \left\{ \left[ \frac{d}{dx}(x) \right] (1 + \ln x) + x \left[ \frac{d}{dx}(1 + \ln x) \right] \right\} \\ &= -\frac{1}{x^2(1 + \ln x)^2} \left[ (1)(1 + \ln x) + x \left( \frac{1}{x} \right) \right] \\ &= -\frac{\ln x + 2}{x^2(1 + \ln x)^2} \end{aligned}$$